take derivative of this

Determine the maximum and minimum velocity of the function given

2) $\boxed{v(t) = t^3 - 3t^2 + 4}$ [0, 4]

Absolute Extrema

$v'(t) = 3t^2 - 6t$

$0 = 3t^2 - 6t$

$0 = 3t(t-2)$

$t = 0 \quad t = 2$

$v(0) = 4$

$v(2) = 0$ min at $(2, 0)$

$v(4) = 20$ max at $(4, 20)$

Take derivative of this

Determine the maximum and minimum acceleration of the function given

absolute extrema

5) $v(t) = 4t^2 - 6t^3$ [0, 3]

$\boxed{a(t) = 8t - 18t^2}$

$a'(t) = 8 - 36t$

$0 = 8 - 36t$

$36t = 8$

$t = \frac{8}{36} = \frac{2}{9}$

$a(0) = 0$

$a(2/9) = 8\left(\frac{2}{9}\right) - 18\left(\frac{2}{9}\right)^2 =$

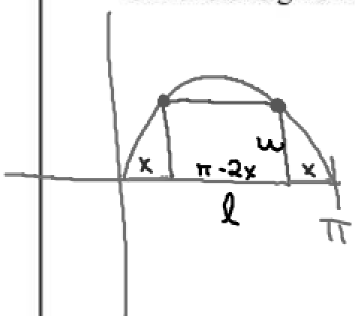
$a(3) = 24 - 162 =$

What you'll Learn About:
How to use derivatives to solve real world problems

and touches the x-axis

A) A rectangle is to be inscribed under one arch of the sine curve. What is the $\boxed{\text{largest area}}$ the rectangle can have, and what dimensions give that area. → Maximize the Area

↑ Take derivative of Area



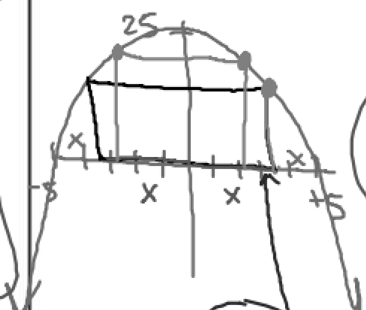
$A = lw$

$l = \pi - 2x$

$w = \sin x$

$A = \sin x(\pi - 2x)$

B) A rectangle is to be ~~inscribed~~ between the curve $y = 25 - x^2$ and the x-axis. What is the largest area the rectangle can have, and what dimensions give that area.



$x = \frac{5}{\sqrt{3}}$

$A = lw$

$l = 2x$

$w = 25 - x^2$

$A = 2x(25 - x^2)$

$A = 50x - 2x^3$

$\boxed{A'(x) = 50 - 6x^2}$

$0 = 50 - 6x^2$

$\frac{6x^2}{6} = \frac{50}{6}$

$x^2 = \frac{25}{3}$

$x = \frac{\pm 5}{\sqrt{3}}$

$$\boxed{A = \frac{10}{\sqrt{3}}\left(25 - \left(\frac{5}{\sqrt{3}}\right)^2\right)}$$

$$\boxed{\begin{aligned} l &= 2\left(\tfrac{5}{\sqrt{3}}\right) \\ w &= 25 - \left(\tfrac{5}{\sqrt{3}}\right)^2 \end{aligned}}$$

1st deriv

$A'(1) = 44 > 0$

$A'(4) = -46 < 0$

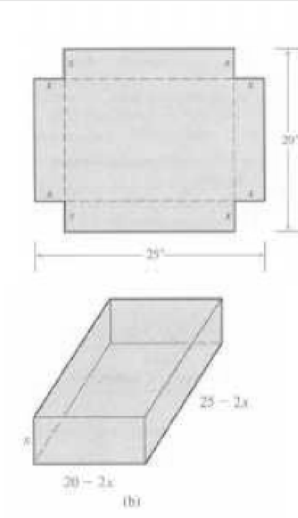
$x = \frac{5}{\sqrt{3}}$ is a local max b/c f' changes from + to −

2nd deriv test

$A''(x) = -12x$

$A''\left(\frac{5}{\sqrt{3}}\right) = -12\left(\frac{5}{\sqrt{3}}\right) < 0$

$x = \frac{5}{\sqrt{3}}$ is local max

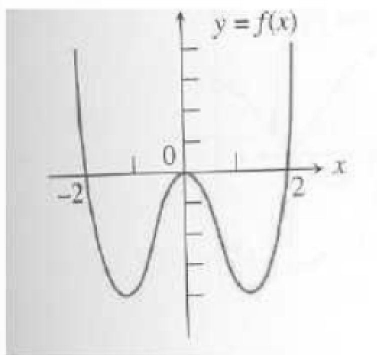


An open-top box is to be made by cutting congruent squares of side length x from the corners of a 20 by 25 inch sheet of tin and bending up the sides. How large should the squares be to make the box hold as much as possible? What is the resulting volume?

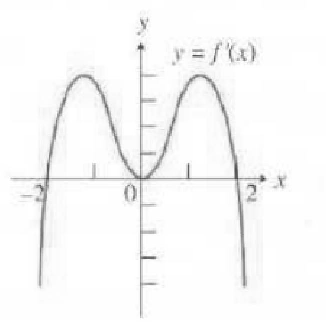
You have 40 feet of fence to enclose a rectangular garden along the side of a barn. What is the maximum area that you can enclose?

What you'll Learn About:
How to interpret graphs of f(x), $f'(x)$, and $f''(x)$

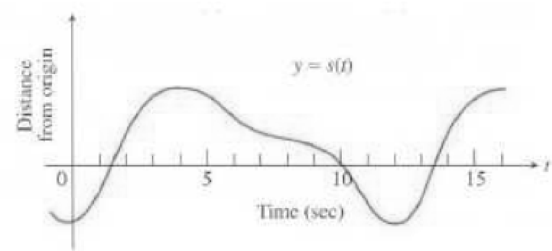
22) Use the graph of the function f to estimate where
a) $f' = 0$ b) $f' > 0$ c) $f' < 0$ d) $f'' = 0$ e) $f'' > 0$ f) $f'' < 0$



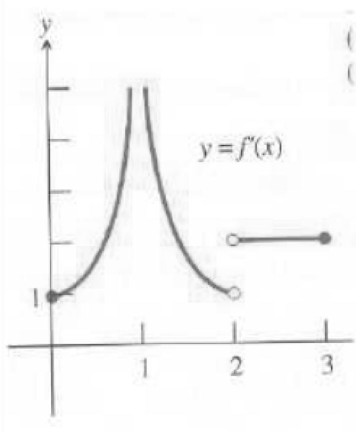
22) Use the graph of the function f' to estimate the intervals on which
a) f is increasing b) f is decreasing c) f is concave up d) f is concave down
and then use the graph of the function f' to find
e) any extreme values and f) any points of inflection



30) Using the graph of the position function find the approximate values at which v(t) = 0 and when a(t) = 0.



50) Use the graph of the function f' to estimate the intervals on which
a) f is increasing b) f is decreasing c) f is concave up d) f is concave down
and then use the graph of the function f' to find
e) any extreme values and f) any points of inflection
(Assume that the function f is continuous from [0, 3]



What you'll Learn About:
How to sketch graphs of f(x), $f'(x)$, and $f''(x)$

40a) $f(2) = 3$

$f'(2) = 0$

$f'(x) > 0$ for $x < 2$

$f'(x) < 0$ for $x > 2$

40d) $f(2) = 3$

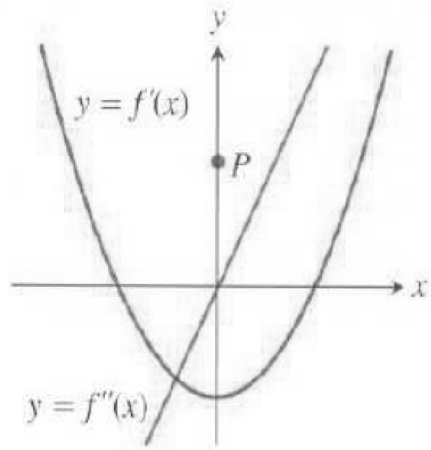
$f'(2) = 0$

$f'(x) > 0$ for $x \neq 2$

What you'll Learn About:
How to sketch graphs of f(x), $f'(x)$, and $f''(x)$

Sketch a possible graph of f(x) that passes through point P

41.



Sketch a continuous curve with the following properties

$f(-8) = 0$

$f(-4) = 2$

$f(8) = 4$

$f'(8) = f'(-8)$

$f'(x) > 0 \quad |x| < 8$

$f'(x) < 0 \quad |x| > 8$

$f''(x) > 0 \quad x < 0$

$f''(x) < 0 \quad x > 0$

Sketch a continuous curve

x	Y	Curve
x < -2		Increasing, concave down
-2	1	Horizontal tangent
-2 < x <1		Decreasing, concave down
1/2	-1	Inflection Point
1	-4	Horizontal Tangent
1 < x < 3		Increasing, concave up
3	5	Inflection Point
4	7	Horizontal tangent
x > 4		Increasing, concave up

Sketch a continuous curve if the function below is an even function that is continuous on [-3 , 3]

x	0	1	2	3
f	0	2	0	-2
f'	3	0	DNE	-3
f''	0	-1	DNE	0

x	$0 < x < 1$	$1 < x < 2$	$2 < x < 3$
f	+	+	-
f'	+	-	-
f''	-	-	-

Summary of interpreting Graphs

		Given graph of f(x)	Given graph of $f'(x)$	Given graph of $f''(x)$
f(x) has a Critical Point	$f'(x)=0$ $f'(x)$ Und	Slope of f(x) = 0 Look for max/mins of f(x)	Find the x-intercepts of the $f'(x)$ graph	X
f(x) increasing	$f'(x)>0$	Slope of f(x) is positive Look where f(x) is increasing	Find where the $f'(x)$ graph is above the x-axis	X
f(x) decreasing	$f'(x)<0$	Slope of f(x) is negative Look where f(x) is decreasing	Find where the $f'(x)$ graph is below the x-axis	X
f(x) has a possible inflection point	$f''(x)=0$ $f''(x)$ Und	Trace f(x) see where the concavity changes	Find where the $f'(x)$ graph changes slope These should be the max and mins of the $f'(x)$ graph	Find the x-intercepts of the $f''(x)$
f(x) is concave up	$f''(x)>0$	Trace f(x) see when the graph is concave up	Find where the $f'(x)$ graph is increasing Slope of $f'(x)>0$	Find where the graph of $f''(x)$ is above the x-axis
f(x) is concave down	$f''(x)<0$	Trace f(x) see when the graph is concave down	Find where the $f'(x)$ graph is decreasing Slope of $f'(x)<0$	Find where the graph of $f''(x)$ is below the x-axis
Local Maximum		Look where the graph of f(x) changes from increasing to decreasing	Look where the graph of $f'(x)$ crosses the x-axis and moves from above to below the axis	X
Local Minimum		Look where the graph of f(x) changes from decreasing to increasing	Look where the graph of $f'(x)$ crosses the x-axis and moves from below to above the axis	X
Points of Inflection		Trace f(x) see where the concavity changes	Find where the $f'(x)$ graph changes slope These should be the max and mins of the $f'(x)$ graph	Find where the graph of $f''(x)$ crosses the x- axis and moves from above to below the x-axis or below to above the axis

Summary of Characteristics of graphs

If f' is undefined or if $f' = 0$, this is a Critical Point (Possible Local Max or Min)
If $f' > 0$ the original function f is increasing
If $f' < 0$ the original function f is decreasing

If f'' is undefined or if $f'' = 0$, this is a possible Inflection Point (Change in concavity)
If $f'' > 0$ the original function f is concave up
If $f'' < 0$ the original function f is concave down

Anytime the graph changes concavity you have an inflection point

To find intervals of increase and decrease

1. Find the first derivative
2. Set the first derivative equal to zero (These will be your critical points)
   - Don't forget to check when the first derivative is undefined
3. Pick values to the left and right of your critical points
4. If $f' > 0$ the original function f is increasing
5. If $f' < 0$ the original function f is decreasing

To find intervals of concavity

1. Find the second derivative
2. Set the second derivative equal to zero (These are your possible inflection points)
   - Don't forget to check when the second derivative is undefined
3. Pick values to the left and right of your possible inflection points
4. If $f'' > 0$ the original function f is concave up
5. If $f'' < 0$ the original function f is concave down
6. If f'' changes sign that is a point of inflection

To find a local/relative maximum or local/relative minimum

Use the first derivative test

1. If your original function changes from increasing to decreasing you have a local maximum
2. If your original function changes from decreasing to increasing you have a local minimum

Use the 2^{nd} derivative test

1. Plug your critical points into your second derivative
2. If your original function is concave up at the critical point, the critical point is a local min
3. If your original function is concave down at the critical point, the critical point is local max

Absolute Max/Min

1. Plug your critical points and your endpoints back into the original equation
2. The biggest value is your absolute max
3. Your smallest value is your absolute min